Christ Church Grammar School

2019
TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(4 marks)**

Write the following equations without logarithms:

(a) $logA = logb + 2logC$ (2 marks)

(b) $lnM = 3\,ln(a) - 2$ (2 marks)

See next page

Question 2 **(7 marks)**

(a) Calculate each of the following definite integrals, simplifying your answers.

(i) $\int_0^1 x^2 e^{x^3} dx$ (2 marks)

(ii) $\int_1^2 \frac{2x^3+1}{x^4+2x} dx$ (3 marks)

(b) Simplify 2019^{w}, where $w = \frac{1}{\ln 2019}$ (2 marks)

Question 3 **(4 marks)**

The gradient function of a curve is given by $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$. Find the equation of the curve given that it passes through the point $(0, -2)$.

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

2019
TEST 4

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

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Question 4 **(5 marks)**

Suppose that a random variable X has the probability density function with rule:

$$f(x) = \begin{cases} cx, & if\ 0 \leq x \leq 2 \\ 0, & if\ x > 2\ or\ x < 0 \end{cases}$$

(a) Find the value of c that makes f a probability density function. (2 marks)

(b) Find $\Pr(X > 1.5)$. (2 marks)

(c) Find $\Pr(1 \leq X \leq 1.5)$. (1 mark)

Question 5 **(4 marks)**

It is known that 42% of students studying medicine at UWA have a parent who is a doctor of medicine. A sample of 50 students studying medicine at UWA was taken. Describe the distribution of the sample proportion and estimate the probability that the sample proportion of students with parents who are doctors of medicine is no more than 0.5.

See next page

Question 6 **(6 marks)**

The number of customers entering Aldi's per day is modelled by a normal random variable with a mean of 350 and a standard deviation of 18.

(a) Determine the probability that tomorrow there will be:

(i) less than 340 customers. (1 mark)

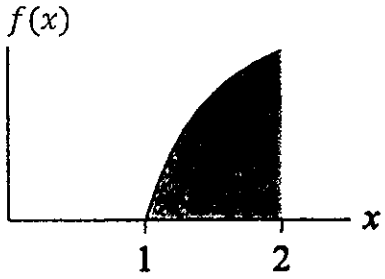
(ii) less than 370 given that there are more than 340 customers. (2 marks)

(b) Determine the probability that over the next 5 days there will be between 340 and 370 customers on exactly 3 of those days. Show all distributions and relevant parameters that you use. (3 marks)

Question 7 **(3 marks)**

The equation and graph of a probability density function is shown below. Find the value of k, clearly showing your method.

$$f(x) = \begin{cases} k - \frac{k}{x^2}, & 1 \leq x \leq 2 \\ 0 \quad , & elsewhere \end{cases}$$



Question 8 **(2 marks)**

A-Max Finance has 49 office staff, 28 store workers and 21 delivery drivers. How many of each should be selected to make a stratified random sample of 10?

See next page

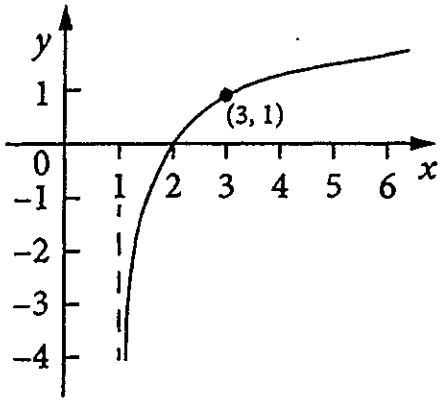
Question 9 **(2 marks)**

In the case below, state whether or not the sampling method is fair, and if it isn't, state one kind of bias involved.

A reality TV show eliminates one contestant each week by having people SMS their choice of who gets eliminated to a particular number each week. They have the system set up so that only one vote is accepted for each mobile number.

Question 10 **(3 marks)**

The function f has a domain $(1, \infty)$ and its graph is as shown below. Given that $y = \log_a(x + b)$, find the equation of this function.



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Question 11 **(5 marks)**

The random variable X has a probability density function $f(x) = \frac{\sqrt{x}}{18}$ for $0 \leq x \leq 9$.

(a) Find the mean μ and the standard deviation σ for X. (2 marks)

(b) Find the mean μ for T if $T = 2X + 5$. (1 mark)

(c) Find the standard deviation σ for P if $P = \frac{X-\mu}{\sigma}$. (2 marks)

End of questions

Christ Church Grammar School

2019
UNIT TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Student name _____

Teacher name - SOLUTIONS -

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Question 1 **(4 marks)**

Write the following equations without logarithms:

(a) $\log A = \log b + 2\log c$ (2 marks)

$\log A = \log b + \log c^2$ ✓ Uses log law

$\log A = \log bc^2$

$\therefore A = bc^2$ ✓ Simplifies

(b) $\ln M = 3\ln a - 2$ (2 marks)

$\ln M = \ln a^3 - \ln e^2$ ✓ Uses log law

$\ln M = \ln \frac{a^3}{e^2}$

$\therefore M = \frac{a^3}{e^2}$ ✓ Simplifies

4

Question 2 **(7 marks)**

(a) Calculate each of the following definite integrals, simplifying your answers.

(i) $\int_0^1 x^2 e^{x^3} dx$ (2 marks)

$\frac{1}{3}\int_0^1 3x^2 e^{x^3} dx = \left[\frac{1}{3}e^{x^3}\right]_0^1$ ✓ (Integrates)

$= \frac{1}{3}e - \frac{1}{3}$

$= \frac{1}{3}(e-1)$ ✓ (Answ. must be factorised)

(ii) $\int_1^2 \frac{2x^3+1}{x^4+2x} dx$ (3 marks)

$\frac{1}{2}\int_1^2 \frac{4x^3+2}{x^4+2x} dx$

$= \frac{1}{2}\left[\ln|x^4+2x|\right]_1^2$ ✓ (Integrates)

$= \frac{1}{2}\left[\ln 20 - \ln 3\right]$ ✓ (substitutes)

$= \frac{1}{2}\ln\left(\frac{20}{3}\right)$ ✓ (simplifies)

(b) Simplify $2019^{\frac{1}{\ln 2019}}$ (2 marks)

$y = 2019^{\frac{1}{\ln 2019}}$

$\ln y = \frac{1}{\ln 2019} \cdot \ln 2019$ ✓ (uses log laws)

$\ln y = 1$

$\therefore y = e$ ✓ (simplifies)

See next page

7

Question 3 **(4 marks)**

The gradient function of a curve is given by $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$. Find the equation of the curve given that it passes through the point $(0, -2)$.

If $\frac{dy}{dx} = e^x + \frac{x}{1+x^2} - 1$

then $y = e^x + \frac{1}{2}\ln|1+x^2| - x + c$ ✓✓ (Anti-diff)

But when $x = 0$, $y = -2$.

$\therefore -2 = e^0 + \frac{1}{2}\ln 1 - 0 + c$

$-2 = 1 + 0 - 0 + c$

$\therefore c = -3$ ✓ (c-value)

$\therefore$ Equation is $y = e^x + \frac{1}{2}\ln|1+x^2| - x - 3$ ✓ (Equ)

4

End of questions

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Section Two:
Calculator-assumed

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Question 4 **(5 marks)**

Suppose that a random variable X has the probability density function with rule:

$$f(x) = \begin{cases} cx, & if\ 0 \le x \le 2 \\ 0, & if\ x > 2\ or\ x < 0 \end{cases}$$

(a) Find the value of c that makes f a probability density function. (2 marks)

Since f is a pdf $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$\therefore \int_0^2 cx\,dx = 1$ ✓ Integral statement

$\therefore 2c = 1$

$c = \frac{1}{2}$ ✓ Answer.

(b) Find $\Pr(X > 1.5)$. (2 marks)

$\int_{1.5}^{2} 0.5x\,dx$ ✓ (correct limits of integration)

$= 0.4375$ ✓ Answ. $\frac{7}{16}$

(c) Find $\Pr(1 \le X \le 1.5)$. (1 mark)

$\int_1^{1.5} 0.5x\,dx = 0.3125$ ✓ $\frac{5}{16}$

5

Question 5 **(4 marks)**

It is known that 42% of students studying medicine at UWA have a parent who is a doctor of medicine. A sample of 50 students studying medicine at UWA was taken. Describe the distribution of the sample proportion and estimate the probability that the sample proportion of students with parents who are doctors of medicine is no more than 0.5.

By CLT (sample > 30) (states Normal) ✓

Distribution ~ Normal with

mean = 0.42 & $\sigma = \sqrt{\frac{0.42(1-0.42)}{50}}$

$\sigma = 0.06980$

✓ mean
✓ SD

$\therefore P(\hat{\pi} \leq .50) \approx 0.87413$ ✓ (Prob)

4

Question 6 **(6 marks)**

The number of customers entering Aldi's per day is modelled by a normal random variable with a mean of 350 and a standard deviation of 18.

(a) Determine the probability that tomorrow there will be:

(i) less than 340 customers. (1 mark)

$P(X < 340) = 0.2893$ ✓

(ii) less than 370 given that there are more than 340 customers. (2 marks)

$$P(X < 370 \mid X > 340) = \frac{P(340 < X < 370)}{P(X > 340)}$$

$$= \frac{0.57748}{0.71074}$$ ✓ correct denominator

$$= 0.8125$$ ✓

(b) Determine the probability that over the next 5 days there will be between 340 and 370 customers on exactly 3 of those days. Show all distributions and relevant parameters that you use. (3 marks)

Binomial Dist : $Bin(n, p)$

(✓ correct Dist.)

$Bin(5, 0.57748)$ ✓ Parameters

$= 0.3438$ ✓

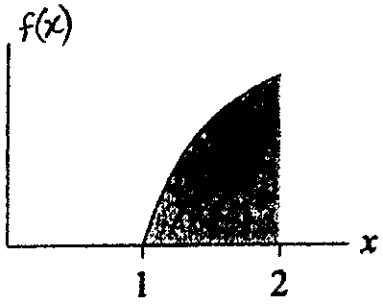
6

See next page

Question 7 **(3 marks)**

The equation and graph of a probability density functions is given. Find the value of k, clearly showing your method.

$$f(x) = \begin{cases} k - \frac{k}{x^2}, & 1 \le x \le 2 \\ 0, & elsewhere \end{cases}$$



(limits) ✓ (equates to 1) ✓

$$\int_1^2 k - \frac{k}{x^2}\, dx = 1 \quad \text{(solve on ClassPad)}$$

$$\therefore \boxed{k = 2} \checkmark$$

Question 8 **(2 marks)**

A-Max finance has 49 office staff, 28 store workers and 21 delivery drivers. How many of each should be selected to make a stratified random sample of 10?

$49 + 28 + 21 = 98$ staff in total ✓ (total 98)

$\therefore$ Office Staff : $\frac{49}{98} \times 10$ $= 5$

Store workers : $\frac{28}{98} \times 10$ $= 3$ ✓ (All must be correct)

Delivery Drivers : $\frac{21}{98} \times 10$ $= 2$

10

5

See next page

Question 9 **(2 marks)**

In the case below, state whether or not the sampling method is fair, and if it isn't, state one kind of bias involved.

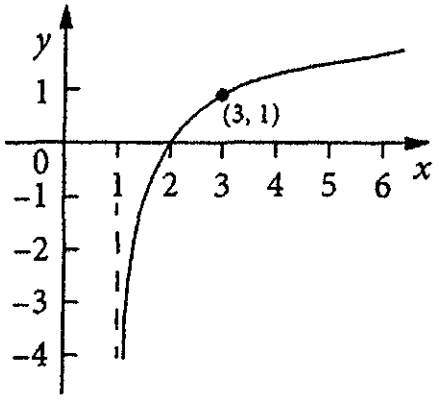
A reality TV show eliminates one contestant each week by having people SMS their choice of who gets eliminated to a particular number each week. They have the system set up so that only one vote is accepted for each mobile number.

Not fair - Bias ✓ (Bias)

Only attracts those interested in the show. ✓ (Reason)
They may influence their friends.

Question 10 **(3 marks)**

The function f has a domain $(1, \infty)$ and its graph is as shown below. Given that $y = \log_a(x + b)$, find the equation of this function.



$y = \log_a(x-1)$ ✓ (shift right b-value)

sub in (3,1)

$1 = \log_a(3-1)$

$1 = \log_a 2$

$\therefore a = 2$ ✓ (a value)

hence $y = \log_2(x-1)$ ✓ (equation)

5

See next page

Question 11 **(5 marks)**

The random variable X has a probability density function $f(x) = \frac{\sqrt{x}}{18}$ for $0 \leq x \leq 9$.

(a) Find the mean μ and the standard deviation σ for X. (2 marks)

$\mu = \int_0^9 x \times \frac{\sqrt{x}}{18}\, dx = 5.4$ ✓ (mean)

$\sigma^2 = \int_0^9 (x-5.4)^2 \times \frac{\sqrt{x}}{18}\, dx = 5.554 \left(\frac{972}{175}\right)$

$\therefore \sigma = 2.3568 \left(\frac{18\sqrt{21}}{35}\right)$ ✓ (σ)

(b) Find the mean μ for T if $T = 2X + 5$. (1 mark)

$\mu_T = 2(5.4) + 5$

$\mu_T = 15.8 \left(\frac{79}{5}\right)$ ✓

(c) Find the standard deviation σ for P if $P = \frac{X-\mu}{\sigma}$. (2 marks)

$\sigma_P = \frac{X}{\sigma} - \frac{\mu}{\sigma}$

$\sigma_P = \frac{1}{\sigma} X \quad \therefore \quad \frac{18\sqrt{21}}{35} \times \frac{35}{8\sqrt{21}}$

(USES CHANGE OF SCALE ONLY) ✓ ie $\sigma_P = 1$ ✓ (ANSW)

5

End of questions

Additional working space

Question number: _____

See next page